Exercise 2: Applications of nuclear chemistry

The text details the proton-proton cycle in which fusion in the Sun’s core results in the generation of $^4_2\text{He}$ nuclei. However, after 5 billion years or so from now, the Sun’s core will run out of sufficient number of protons to fuse. At that point, the core will compress due to the enormous gravitational force of the Sun’s mass and the temperature (yes, it’s almost the ideal gas law-type of compressional heating here) of the core will rise from the 4.0 ¥ 10^7 K needed for proton-proton fusion to 1.0 ¥ 10^8 K. At that point, the triple-alpha process will fuse helium. First, two alpha particles fuse (predict the product):

$$\begin{array}{c}
2\text{He} + 2\text{He} \\
\rightarrow
\end{array}$$

2. Then write the balanced equation for the fusion of another alpha particle with the product of the first reaction. This reaction also emits a gamma ray.

3. a. Using the binding energy graph on the next page, estimate and calculate the nuclear binding energy (NBE) change in the fusion of 4 hydrogen nuclei to 1 helium nucleus, and for the fusion of 3 helium nuclei to 1 carbon nucleus.

b. Which process will net more energy production per complete reaction?

c. When the Sun starts to use the triple-alpha process, will there be more fusion reactions, the same number or fewer reactions than when the Sun was performing proton-proton fusion? Explain your answer. Hint: The Sun must stay inflated!
The Sun will cease all fusion once the helium nuclei supply is exhausted, and will collapse into a white dwarf made of degenerate carbon nuclei floating in an electron gas. For stars slightly more massive than the Sun, the additional gravity will allow compression of the core so that temperatures of $5.0 \times 10^8$ K can be achieved.

4. a. At that temperature, two carbon-12 nuclei will fuse. Write the balanced reaction for this fusion, and include an emitted gamma ray.

b. The product of that reaction immediately undergoes a decay into sodium-23. Write the balanced reaction for this decay (what is the other product?).

c. But the products of the decay above immediately recombine and form two products, one of which is an alpha particle. Write the balanced reaction for this reaction.
d. Which recombine and form two further products, one of which is a neutron. Write the balanced reaction for this reaction.

e. Finally, those products recombine and form three products, two of which are alpha particles. Write the balanced reaction for this reaction.

5. **Estimate and calculate** the nuclear binding energy (NBE) change in the fusion for the carbon to oxygen fusion process (one carbon nucleus becomes one oxygen nucleus). How does this compare with the proton-proton cycle and the triple-alpha process? What trend is shaping up as heavier and heavier nuclei are being used in the fusion?

6. The formula used to determine a numerical age for a particular isotope is:

\[
 t = \frac{1}{\lambda} \ln \left( \frac{N_0}{N} \right)
\]

where \( t \) is the age of the sample in years, \( \lambda \) is a constant related to the half-life of the radioactive isotope used (\( \lambda = 0.693 / t_{1/2} \)), \( N \) is the number of atoms of that radioactive isotope in the sample now, and \( N_0 \) is the number of atoms of that radioactive isotope when the sample crystallized (this is generally calculated by measuring the amount of daughter product). For radiometric dating using uranium-238 as the parent isotope, \( \lambda = 1.54 \times 10^{-10} \) yr\(^{-1} \). What is the half-life of U-238? Show your calculation!
7. **Calculate the numerical age** for volcanic rock X and plutonic rock Z and fill in the table below.

<table>
<thead>
<tr>
<th>Rock sample</th>
<th>Number of uranium atoms originally ( (N_0) )</th>
<th>Number of uranium atoms in sample ( (N) )</th>
<th>Calculated numerical age in years ( (t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volcanic rock X</td>
<td>10000</td>
<td>9667</td>
<td></td>
</tr>
<tr>
<td>Plutonic rock Z</td>
<td>10000</td>
<td>9940</td>
<td></td>
</tr>
</tbody>
</table>

8. What *event* do the ages in question 7 represent?

9. The following table gives the quality factor \( (Q) \) for different kinds of ionizing radiation:

<table>
<thead>
<tr>
<th>Radiation source</th>
<th>Alpha particles</th>
<th>High-energy neutrons</th>
<th>Beta particles</th>
<th>Gamma rays</th>
<th>X-rays</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q )</td>
<td>10</td>
<td>10</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

a. What does the information in this table suggest are the most human-harmful types of radiation?

b. So which is more harmful to you: exposure to 10 millirads of alpha particle radiation from spending a night in a radon gas-infested basement, or exposure to 10 millirads of X-radiation from a dental X-ray?