Homework Guide: Vectors

Vectors: Many quantities used in physics are “vector” quantities. You will be expected to do the following with vector quantities.

- **Draw Vectors:** You should be able to draw vectors from a description of their magnitude and direction. You will also need to give the magnitude and direction from a drawing (using a protractor and ruler). Sometimes you will be asked to “sketch” instead of draw. A sketch is an approximate drawing done without a protractor or ruler.

  **For Example:** Fill in the table

<table>
<thead>
<tr>
<th>Draw or Sketch</th>
<th>Description with Magnitude and Direction (Give additional descriptions that are different but equivalent.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$A = 76.4$ miles @ $33^\circ$ N of W $A = 76.4$ miles @ $57^\circ$ W of N $A = 76.4$ miles @ $147^\circ$</td>
</tr>
<tr>
<td>$B = 30$ miles @ $7^\circ$ below the $-x$ axis</td>
<td></td>
</tr>
</tbody>
</table>

- **Find the components of a vector:** The components will need to be reported in table format as shown below. To find the components, it is best to make a sketch of your vector first. Then draw in a “right-triangle” with the vector as the hypotenuse. The “legs” of the triangle will be parallel to the coordinate axes. Then use your trigonometry to solve for the components.

<table>
<thead>
<tr>
<th>“x” Component</th>
<th>“y” Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A = 76.4$ miles @ $57^\circ$ W of N</td>
<td>This component will be negative because it is pointing left. $A_x = 76.4$ miles (cos $33^\circ$) $A_x = -64.1$ miles</td>
</tr>
<tr>
<td></td>
<td>This component will be positive because it is pointing up. $A_y = 76.4$ miles (sin $33^\circ$) $A_y = +41.6$ miles</td>
</tr>
</tbody>
</table>
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- **Add vectors graphically (by construction):** You will be expected to be able to use a ruler and a protractor to add vectors by drawing. Vectors are added “head-to-tail”. This means that when you finish drawing the first vector, you add the second vector’s tail to the head of the first. If we practice drawing the addition as the equation states we will have everything in order. \( A + B \), first draw vector \( A \) then + (glue) then vector \( B \). Vector \( A = 76.4 \text{ miles} @ 57^\circ \text{ W of N} \) and vector \( B = 30 \text{ miles} @ 7^\circ \text{ below the } -x \text{ axis} \). Find \( A + B \) graphically.

Note: The result of a vector addition is called the resultant.

**Step 1:** Label the origin of your coordinate system “start”.

**Step 2:** Decide on a scale for drawing the length of your vectors. Indicate this on your drawing.

**Step 3:** Draw vector \( A \) on the coordinate system with its tail at the origin. Be sure to draw its “head”. You should have

**Step 4:** Draw some “glue” (on the drawing it looks like a light gray blob) onto the head of vector \( A \). This “glue” will let us “add” vector \( B \) to vector \( A \).

**Step 5:** Now put a new coordinate system at the head of vector \( A \) and draw vector \( B \) here. B’s tail will now be glued to A’s head.

**Step 6:** Repeat Steps 3 and 4 until you have added all the vectors.

**Step 7:** Label the head of the last vector “end” as you have just ended the “adding” process.

**Step 8:** Draw your resultant from the “start” to the “end”. That means that the “head” of the resultant will be at the “end”. Measure the magnitude and direction for this vector. Convert its length as drawn with your scale and report your result (both magnitude and direction).

**Sample Question for Practice:**
Graphically added the following vectors to find \( A + B + C \)

\[ A = 66.5 \text{ miles/hr} @ 67^\circ \text{ E of N} \]
\[ B = 110 \text{ miles/hr} @ -20^\circ \]
\[ C = 25.3 \text{ miles/hr} @ 123^\circ \]

What is your scale? 1cm = ________ miles/hr

(ans: 151 mi/hr @ 3.6°)

PHY 101: Homework Guide
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Add vectors by components: Begin by filling out the Component table for each vector. You will notice below that for vectors A and B, the components are in the table in their respective column. Make sure to include the signs!

Then you will need to add the components to get an x-component for the sum and a y-component for the sum. These components will then be used to find the magnitude and direction of the resultant.

<table>
<thead>
<tr>
<th>“x” Component</th>
<th>“y” Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>This component will be negative because it is pointing left.</td>
<td>This component will be positive because it is pointing up.</td>
</tr>
<tr>
<td>A = 76.4 miles @ 57° W of N</td>
<td>A = 76.4 miles (cos 33°)</td>
</tr>
<tr>
<td>$A_x = 76.4$ miles &lt;sup&gt;−&lt;/sup&gt; 64.1 miles</td>
<td>$A_y = 76.4$ miles (sin 33°)</td>
</tr>
<tr>
<td>$A_x = - 64.1$ miles</td>
<td>$A_y = + 41.6$ miles</td>
</tr>
<tr>
<td>This component will be negative because it is pointing left.</td>
<td>This component will be negative because it is pointing down.</td>
</tr>
<tr>
<td>B = 30 miles @ 7° below the −x axis</td>
<td>B = 30 miles (cos 7°)</td>
</tr>
<tr>
<td>$B_x = 30$ miles &lt;sup&gt;−&lt;/sup&gt; 29.8 miles</td>
<td>$B_y = 30$ miles (sin 7°)</td>
</tr>
<tr>
<td>$B_x = - 29.8$ miles</td>
<td>$B_y = - 3.7$ miles</td>
</tr>
<tr>
<td>A + B (Also called the resultant of A + B)</td>
<td>- 64.1 miles - 29.8 miles</td>
</tr>
<tr>
<td>- 93.9 miles</td>
<td>+ 41.6 miles – 3.7 miles</td>
</tr>
<tr>
<td>+ 37.9 miles</td>
<td></td>
</tr>
</tbody>
</table>

Remember that you have NOT FINISHED until you are able to give both the magnitude and direction of the resultant. I recommend sketching (or drawing) your vector resultant to see what the direction will look like. You have the x and y-components to help you do the sketch.

The vector is the hypotenuse of a right triangle that has sides that are the x-component and the y-component. Use this with the Pythagorean theorem to find the magnitude of the resultant.

\[ a^2 + b^2 = c^2 \]
\[ (93.9 \text{ miles})^2 + (37.9 \text{ miles})^2 = c^2 \]
\[ 10254 \text{ miles}^2 = c^2 \]
\[ \sqrt{(10254) \text{ miles}^2} = c \]
\[ 101 \text{ miles} \]

Now we need to find the direction. We will use the tangent function to do that:

\[ \tan^{-1}\left(\frac{37.9}{93.9}\right) = \tan^{-1}(0.404) = 22° \]

You must notice (with your eyes, not with your calculator) that the angle is above the −x axis. So the solution is:

\[ A+B=101 \text{ miles @ 22° above the −x axis.} \]
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✔️ Check your work (adding graphically)

- Do you know how to use a protractor?
- Did you draw all vectors with heads?
- Did you include glue, “start”, “end”, and coordinate systems for each vector drawn?
- Did you convert the length of your resultant (probably in cm) to the proper units using your scale?
- Did you include your scale in your drawing?
- Did you give your answer (resultant) with Magnitude and Direction? Giving only the components of a resultant is not enough.
- Did you include units with your magnitude?
- Did you think about whether your direction angle needs a “from” designation such as “below the –x axis” or “S of W” or a “negative sign” to indicate Clockwise from the standard 0º location?

✔️ Check your work (adding by components)

- Did you remember the signs with your components?
- Did you put everything in a table with the rows and columns labeled? Make sure they are in the same order and format as shown in the example above.
- Did you give your answer (resultant) with Magnitude and Direction? Giving only the components of a resultant is not enough.
- Did you include units with your magnitude?
- Did you think about whether your direction angle needs a “from” designation such as “below the –x axis” or “S of W” or a “negative sign” to indicate Clockwise from the standard 0º location?