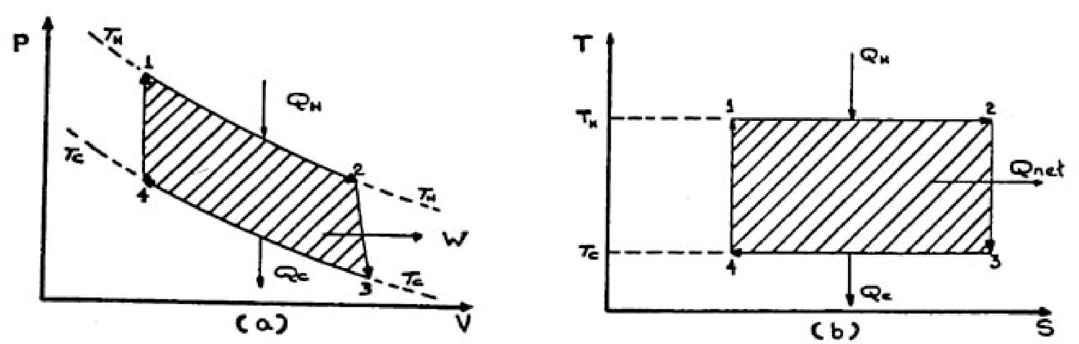
Chemistry 150, Winter 2007 Name:

Exercise 3: The Carnot Cycle

The **Carnot cycle** was developed by Sadi Nicolas Leonard Carnot, a French military engineer, in 1824, and is considered the strongest application of the laws of thermodynamics. It is a description of the mechanism of a "heat engine", in which the transfer of heat between two reservoirs of material, one hot, one cold, allow for work to be extracted. This principle underlies all engines, and much work has been done to extract as much useful work as possible from such a system.

The Carnot cycle is illustrated schematically below. Imagine a piston, which has access to a limitless supply of a hot gas with temperature T_H (the temperature of the "hot" reservoir); the outside of the piston is at temperature T_C (the temperature of the "cold" reservoir) and the gas in the piston will cool to that temperature if left alone. The left diagram plots changes in volume inside the piston versus pressure inside the piston during the cycle; the right diagram plots changes in entropy inside the piston versus temperature inside the piston.



The cycle is a four stage reversible sequence that connect the corners of the shaded area:

1 to 2: Isothermal expansion
2 to 3: Adiabatic expansion
3 to 4: Isothermal compression
4 to 1: Adiabatic compression

The word "adiabatic" means that there is no heat transfer at that step, i.e., Q = 0. Also recall that ΔE = internal energy of the system = Q + w, where w is the pressure-volume work done by the system.

1. What is ΔE for one complete cycle (from point 1 around to point 1, let's say)? Why does it have to be this value? Hint: what's a **state function**? What is ΔS for one complete cycle, assuming it, too, is a state function?

2. Examine the left diagram above and explain why step 1 to 2 and step 3 to 4 are called "**isothermal**".

3. Why must **heat** (Q_H in the left diagram) be added in step 1 to 2? Hint: look at the relative changes in P and V.

4. In step 2 to 3, it is evident that energy must be transferred, yet since the step is adiabatic, Q = 0. In what form is the energy being transferred? What sign does this quantity have? Give this quantity the subscript "$_{2 \Rightarrow 3}$".

5. In step 3 to 4, an isothermal step, Q_L is being released from the system; what is its sign compared to Q_H?

6. In step 4 to 1, an adiabatic step, in what form is the energy being transferred? What sign does this quantity have? Give this quantity the subscript "$_{4 \Rightarrow 1}$".

7. Define the change in entropy $\Delta S = Q_{rev}/T$, where Q_{rev} is the heat transferred during any reversible process. Fortunately, all of the steps of the Carnot cycle are reversible. Using the definition of the entropy change for each step that generates entropy, write an equation for ΔS around the entire Carnot cycle path and equate it to the value derived for the total in question 1.

8. Does $Q_H = Q_L$? Explain. Re-express the equality of question 7 such that the term (Q_L/Q_H) shows up.

9. Use the definition of ΔE and the results of questions 1, 4, 6 and 7 to come up with an expression for ΔE and the value that it equals.

10. Separate the work and heat terms; divide through by Q_H and re-express the fraction on the side with the Q terms such that the term (Q_L/Q_H) shows up.

11. Replace the term (Q_L/Q_H) with its equivalent from question 8, and rewrite the equation in question 10.

12. Figure out the physical interpretation of the work terms on the other side of the equation above. What does the whole fraction on that side of the equation mean?

13. (Payoff) If there is no difference in temperature between the two reservoirs, what does the whole fraction equal? What does this mean?

14. (Other payoff) In order for the whole fraction to equal one (what does this mean?), what must T_L be? Is this possible in the real world?