Astronomy 100 Name(s):

Exercise 7: Minimum energy orbit

Though a straight line may be the shortest distance between two points, travelling along the straight line requires a huge expenditure of energy, especially between two planets. In fact, the lowest energy path between two planets is along a **Hohmann transfer orbit**. This orbit is best described in a diagram which you will draw, showing Earth and Jupiter. The ballistic orbit for a trip to Jupiter from Earth would have its **perihelion** on Earth's orbital path (1 AU) and its **aphelion** on Jupiter's orbital path (5.25 AU).

1. On a separate sheet of paper, draw a point at the center of the sheet. This will represent the Sun. Make the assumption that planetary orbits are circular (in this approximation, it's an okay assumption) and, using a scale of 1 AU = 1 inch), draw the orbit of Earth at 1 AU and the orbit of Jupiter at 5.25 AU (all of the orbit won't fit on the paper).

2. What is the shape of a Hohmann transfer orbit (hint: think Kepler's first law)?

3. Next, draw the probe's Hohmann transfer orbit (make it a smooth shape) between Earth's orbit and Jupiter's orbit and highlight the portion of the orbit the probe will use between Earth and Jupiter.

4. What is the **period** of the probe's orbit (hint: use Kepler's third law)? But how long will the **actual flight time** of the probe be (look at the highlighted portion of the orbit)?

5. **Draw** a circle representing Earth where the probe's orbit intersects Earth's orbit and **draw** another circle representing Jupiter where the probe's orbit intersects Jupiter's orbit. You will draw one more Earth and one more Mars, for a total of four bodies drawn. They will be labelled **"Earth at probe launch", "Jupiter at probe launch", "Earth at probe landing"** and **"Jupiter at probe landing"**. **Label** the Earth and Jupiter you have already drawn with the correct labels.

6. (Payoff question) Where should Jupiter be when the probe is launched? **Draw and label it**. Approximately how much of one orbit will the Earth have moved while the probe is in flight? About how much of one orbit will Jupiter have moved during the probe flight?

7. Where will Earth be when the probe lands? **Draw and label it**. Will the Sun get in the way of radio communications with the probe?

The ***vis-viva*** ("see-life") equation was derived from the principle of conservation of energy and connects the speed of an orbiting body at any given point with the distance that it is away from the body it is orbiting:

$$v^2 = G(m_1 + m_2)\left(\frac{2}{r} - \frac{1}{a}\right)$$

where:
- v is the speed of the orbiting body in m/s
- $G = 6.67 \times 10^{-11}$ $m^3/kg \bullet s^2$
- m_1 is the mass of the orbiting object in kg
- m_2 is the mass of the object being orbited in kg
- r is the instantaneous distance between the two objects in m
- a is the semi-major axis of the orbiting object in m

8. Use this equation to calculate the orbital speed needed to keep a satellite with a mass of 200 kg in a **circular** orbit around the earth at a distance of 200 km.

a. For this probe, what **simplification** can you make to the sum ($m_1 + m_2$)? Hint: consider what m_1 and m_2 represent.

b. Using the conversion factor 1 km = 1000 m, calculate the orbital **speed** (in km/s) needed for the probe.

9. a. For the Jupiter probe earlier in the exercise, what object is represented by m_2? So what **simplification** can you make to the sum ($m_1 + m_2$)?

b. For the Jupiter probe leaving Earth orbit, what is r (in AU, then in m)? What is a (in AU, then in m)?

10. Using the *vis-viva* equation, what **speed** (in km/s) must the Jupiter probe achieve in Earth orbit in order to make it to Jupiter? Show the calculation!