Lab 9: Properties of giant planets (and moons)

Masses of the planets

It’s neat how we can know the masses of any of the giant planets; after all, unlike the Earth, there’s no easy way to stand on the surface and take a measurement of surface gravity (in fact, there is no truly solid surface on these planets). Instead, we rely on indirect methods and mathematics. The following equation will tell us the mass of Saturn, for instance, only knowing \( P \), the period of orbit of one of its moons (in seconds) and \( a \), the length of that moon’s semi-major axis (in meters):

\[
M_{\text{Saturn}} = \frac{4\pi^2 a^3}{GP^2}
\]

where \( G \) is the constant of universal gravitation \( = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \).

1. a. Rearrange the equation so that the period is on the opposite side of the equals sign as the semi-major axis.

b. The equation above is just a restatement of what Law?

2. One of the moons of Saturn is Enceladus; its orbital period is 1.370 days and its semi-major axis is 238,000 km.

a. Convert Enceladus’s period to seconds.

b. Convert Enceladus’s semi-major axis to meters.

c. What is the mass of Saturn?

d. Write down the “true” value of Saturn’s mass given in Table A1 of the appendix. How close is your answer to the “true” value?
3. Now that the mass of Saturn is known, you can calculate Saturn’s escape velocity; that is, the minimum speed any object needs to end up in a parabolic or hyperbolic orbit around Saturn.

a. Are parabolic and hyperbolic orbits open or closed curves?

b. Why must we speak of parabolic and hyperbolic orbits, and not simply state that “the escape velocity is the minimum speed of any object needed to leave the planet’s gravity”?

4. a. The formula for calculating the escape velocity is

\[ v_{\text{escape}} = \sqrt{\frac{2GM}{R}} \]

where \( G \) is the constant of universal gravitation, \( M \) and \( R \) are the mass (kg) and radius (m), respectively, of the body that an object is trying to escape. Calculate Saturn’s escape velocity.

b. From a practical fuel-based standpoint why isn’t Cassini coming back to Earth? Or a better question: how would Cassini have to have been modified in order to return to Earth?

As you know, many of the closer moons of the giant planets are subject to tidal heating, the distortion of the mass of the planet by tidal forces in such a way as to generate frictional heat within the body.

5. Play with the units! Tidal heating is reported in terms of J/yr (in other words, an energy per time). If you wanted to calculate the likelihood that tidal heating would melt the rocks of a planet, you’d need to know the power delivered to each kilogram of rocks in the planet; after all, it’s the power sustained over time that will heat and might eventually melt the rocks. Below, just using unit conversions, show how J/yr becomes W/kg. As a hint, \( W = \text{watt} = J/s \).

\[ \frac{J}{\text{yr}} = \frac{W}{\text{kg}} \]

6. The Earth currently generates \( 3.0 \times 10^{19} \) J/yr of tidal heating. The mass of the Earth is \( 6.0 \times 10^{24} \) kg. Calculate the power delivered to each kilogram of rocks on the Earth’s surface.
7. Io, the nearest moon to Jupiter, has 30 times the power delivered per kilogram to its rocks than the Earth does (in other words, thirty times the number you calculated in the previous problem). Io has a mass of \(8.93 \times 10^{22}\) kg. Calculate the rate of tidal heating (J/yr) that Io has. Hint: It’s simply doing the reverse of the problem above, with new numbers.

8. Recall that the effective temperature of a body is the temperature that the surface body needs to be in order to stay at a constant temperature (and not heat up or cool down). The formula for that is:

\[
T_{\text{effective}} = \sqrt[4]{\frac{L}{(5.67 \times 10^{-8})4R^2}}
\]

Note that \(L\) is total power radiated by the body.

a. Calculate \(L\) for Io, using the information from problem 7.

b. Look up \(R\) of Io. Calculate the effective temperature of Io. Remember that the temperature will be in Kelvins (K). Is it hotter than the Earth?

9. The extreme case of tidal heating is when there is so much tidal force exerted on different parts of the moon, the moon breaks apart into several pieces.

a. Does the moon need to be relatively near or relatively far from a planet in order to reach this point? Draw a sketch of the moon that shows the differential forces on the moon (use one set of arrows at the front of the moon and another set at the back of the moon to illustrate your point). A nice site that discusses this is http://pegasus.phast.umass.edu/a100/handouts/roche.html

b. The Roche Limit is the cutoff point for single-body moons to stay intact. What factors besides the difference in tidal forces would affect the distance away from the planet the Roche Limit is.
10. a. The Roche Limit for Saturn is roughly 2.44 planetary radii. Give the distance away from Saturn of its Roche Limit (in km).

b. In the appendix, Table A2, are there any Saturnian moons within the Roche Limit? Yet the rings of Saturn are all within the Roche Limit; how can they exist there?

c. What does this analysis suggest about the origin of Saturn’s rings?

Orbital mechanics
Notice that a critical factor in determining characteristics like planetary masses is the period of its moons. It seems a difficult task, yet even Galileo knew how to calculate the periods of Jupiter’s large moons (now called the Galilean satellites). Here is the analysis:

If \( x = \) the distance from Jupiter in kilometers, then the satellites’ motion is given by the following formula:

\[
x = k \sin \left( \frac{2\pi}{P} t \right)
\]

where \( P \) is the period of the satellite in hours, \( t \) is time since the transit of the satellite (in other words, when the satellite passes between Jupiter and Earth), and \( k \) is the orbital distance of the satellite from Jupiter.

The version of the equation we are going to use, since we want to find the period of the satellites’ orbits is:

\[
P = \frac{2 \sqrt{t}}{\sin^{-1} \left( \frac{x}{k} \right)}
\]

where \( \sin^{-1} \) means “the inverse sine of”. On your calculator, this function can be accessed by hitting the “2nd” or “INV” key prior to hitting the “sin” key.

A. Go to one of the computer labs in the ED building. Alternatively, you can do this on any computer that supports the software.
B. Open a browser and go to:
http://www.gettysburg.edu/academics/physics/clea/juplab.html

C. Download the software (there is a Mac and a PC version). Follow the instructions to run the Jupiter lab software. The most difficult thing is to remember in which directory you put the software.

D. The Jupiter lab software is designed to take students through a calculation of the mass of Jupiter. We will not be using it for that purpose. We will be using it to simulate a series of observations. Go to the “File” heading on the menu bar and select “Log in…”, then, in the window that pops up, type in anything and hit “OK”.

E. Go to “File” on the menu bar again and this time, select “Run”. Another window will open, and will ask for a date and the Universal Time you want the simulation to begin. You will need to convert your Pacific Standard Time into Universal Time: Universal Time uses a 24-hour clock and is set to be coincident with Greenwich Mean Time, which is 8 hours ahead of Pacific Standard Time. Notice, for an observation time of 6:00 p.m. PDT, the Universal Time will be 1:00 a.m. the next day. For instance, the current date and time would be May 25, 2005 at 1:00:00 Universal Time.

F. Click and hold the left mouse button and the cursor will turn into a crosshairs. Drag and center the crosshairs on one of the satellites. In the display at the bottom, there should be a measurement “X=” that gives the satellite-Jupiter distance in units of Jupiter diameters. Record the information below for all four satellites.

11. Below is a picture of Jupiter and its satellites on May 25, 2005 at 1:00:00 Universal Time. Go to “File”, select the “Preferences” choice and then the “Animation” choice from that. This will activate the “Cont.” button on the display. Click this button and watch as the satellites orbit Jupiter. Determine which satellite name corresponds to which lettered body in the picture. Hint: this is not as hard as it sounds, especially if you turn to the satellite data given in the appendix in the text. Click on “Cont.” again to stop the animation.

A = 

B = 

C = 

D =
H. This section will take you through the calculation of the satellite period.

i. Go to “File”, select the “Preferences” choice, and then the “Timing” choice. A window will open; set the “Observation Interval” at 1 hour.

ii. Click the “Next” button on the main display; notice that the satellites move a small distance (in fact, one hour’s worth of movement). Occasionally, you will get a “Sky clouded over; no observation” screen; this is to simulate bad seeing conditions. Ignore them and click “Next”; the sky will clear up.

iii. Choose Io (the closest satellite to Jupiter) and click “Next” until Io is as far away from Jupiter as it is going to get, either to the left of Jupiter or to the right of Jupiter. Record the number of Jupiter diameters away Io is at this point in the table below. Remember that to the left of Jupiter is negative distance and to the right of Jupiter is positive distance. Let’s call this observation time zero hours.

iv. Question 12. Record the distance data for Io over a period of ten hours at hourly intervals; enter this data in Table 9.1. You may miss some due to the “clouded sky” feature. Remember, as Io moves, it may “cross” Jupiter, in which case, the sign of the distance should change. Record for each distance the number of hours (mouse clicks) after the first observation, too.

v. Questions 13 through 15. Repeat i. through iv. for Europa, Ganymede and Callisto.

vi. Fill in table 9.5: k will be the biggest number of Jupiter diameters a given satellite gets away from Jupiter (for the way we set it up, look under the 0 hours entry on the table). Note that k is different for each satellite.

vii. For each satellite pick two (x,t) pairs from the tables and enter this data in table 9.5.

viii. Make sure that your calculator is set on radians (not degrees). Use the second formula on the first page of this exercise (the one that begins “P=”) to calculate the period for each pair (x,t). The calculated period for the same satellite should be roughly the same for both (x,t).

ix. Finally, look up the actual periods of these satellites from the text (note that you may have to convert days to hours!) and answer the questions that follow the table.

Table 9.1: Io orbit simulation observations

<table>
<thead>
<tr>
<th>time (hrs)</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>distance</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 9.2: Europa orbit simulation observations

<table>
<thead>
<tr>
<th>time (hrs)</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>distance (Jup. dia.)</td>
<td></td>
</tr>
</tbody>
</table>

Table 9.3: Ganymede orbit simulation observations

<table>
<thead>
<tr>
<th>time (hrs)</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>distance (Jup. dia.)</td>
<td></td>
</tr>
</tbody>
</table>

Table 9.4: Callisto orbit simulation observations

<table>
<thead>
<tr>
<th>time (hrs)</th>
<th>0</th>
</tr>
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<tbody>
<tr>
<td>distance (Jup. dia.)</td>
<td></td>
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Table 9.5: Calculation of orbital periods

<table>
<thead>
<tr>
<th>Satellite</th>
<th>k (Jupiter diameters)</th>
<th>t (hours)</th>
<th>x (Jupiter diameters)</th>
<th>P (hours)</th>
<th>P (text value in hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Io (short)</td>
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<td></td>
<td></td>
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<tr>
<td>Io (long)</td>
<td></td>
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<tr>
<td>Europa (short)</td>
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</table>
19. You have just completed the same calculation Galileo did four hundred years ago, to show that the moons of Jupiter did indeed orbit Jupiter. Was a conversion to kilometers or AU necessary for any of these calculations you just performed? Short answer: NO. Please explain why the conversion was unnecessary.

20. a. Fill in the lower row of the table below with the percent error of your calculations for the orbital period of the Galilean satellites. To calculate the percent error your values had, use the formula:

\[
\% \text{ error} = \left( \frac{\text{your value} - \text{text value}}{\text{text value}} \right) \times 100.
\]

(Galileo was not a lot more accurate!)

<table>
<thead>
<tr>
<th>Io (short)</th>
<th>Io (long)</th>
<th>Europa (short)</th>
<th>Europa (long)</th>
<th>Ganymede (short)</th>
<th>Ganymede (long)</th>
<th>Callisto (short)</th>
<th>Callisto (long)</th>
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</table>

b. In general, you should have had a correlation: the shorter the observation duration, the worse the percent error. Why do you suppose this might be so?